Energy Conversion – Final Exam

School of Engineering – Dep. of Industrial & Mechanical Eng.

Name: Solution Key
Date: Tuesday, June 4th 2013; 04:00 PM
Location: SCI 402
Instructor: Dr. Wassim Habchi
Notes: No documents allowed
Value: 40% of Total Grade
Time: 3 hours

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Problem I (30 points)

An ideal vapor-compression refrigeration cycle operates with Refrigerant 134a as the working fluid. Saturated vapor enters the compressor at -10°C, and saturated liquid leaves the condenser at 28°C. The mass flow rate of refrigerant is 6 kg/min.

a) Sketch a T-s diagram of this cycle (**5 points**).

b) Determine the specific enthalpy h of the refrigerant at all points in the cycle and the compressor power, in kW (**7 points**).

c) Determine the refrigerating capacity, in kW (**3 points**).

d) Determine the coefficient of performance (**3 points**).

e) Using the definition of the Joule-Thomson coefficient:

$$\mu_{JT} = -\frac{1}{c_p}\left[v - T\left(\frac{\partial v}{\partial T}\right)_P\right]$$

And assuming that the equation of state of refrigerant 134a is:

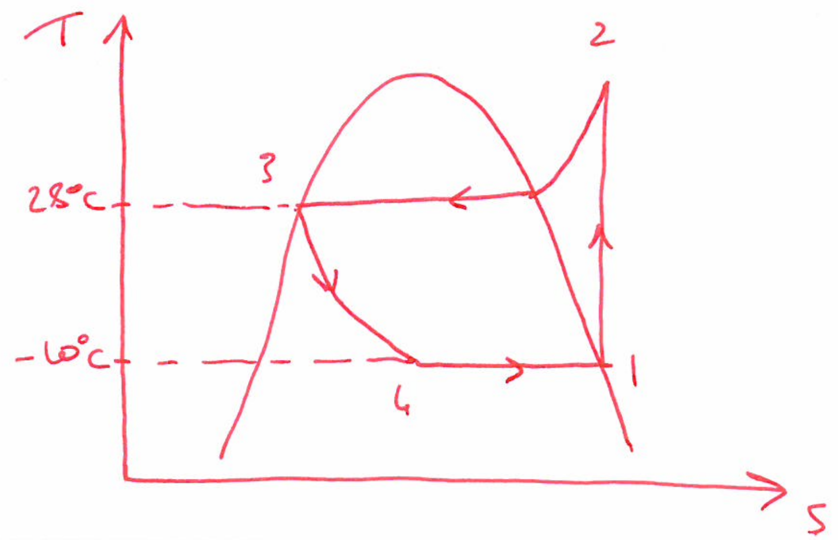
$$P(v+\alpha) = RT$$

Where α is a constant positive number, determine whether the refrigerant temperature will decrease, increase or remain constant during the throttling process (**5 points**).

f) Using Clapeyron equation $\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{T v_{fg}}$, determine the enthalpy of vaporization of the refrigerant at -10°C. Compare the obtained value to the tabulated one (**7 points**).

Solution:

a)



b) • $h_1 = h_g @ -10°C = \boxed{244.51\ kJ/kg}$ & $s_1 = s_g @ -10°C = 0.93766\ kJ/kg.K$

• $h_3 = h_f @ 28°C = \boxed{90.69\ kJ/kg}$ & $P_3 = P_{sat} @ 28°C = 727.31\ kPa$

@②: $P_2 = P_3 = 727.31\ kPa$

$s_2 = s_1 = 0.93766\ kJ/kg.K$

→ @0.7 MPa, & 0.93766 kJ/kg.K → $h = 270.41$ kJ/kg

@0.8 MPa & 0.93766 kJ/kg.K → $h = 273.26$ kJ/kg

$$\Rightarrow h_2 = \frac{0.72731 - 0.7}{0.8 - 0.7} \times (273.26 - 270.41) + 270.41$$

$$\boxed{h_2 = 271.188\ kJ/kg}$$

• $h_4 = h_3 = \boxed{90.69\ kJ/kg}$

$$\Rightarrow \dot{W}_{comp} = \dot{m}(h_2 - h_1) = \frac{6}{60} \times (271.188 - 244.51) = \boxed{2.6678\ kW}$$

c) $\dot{Q}_L = \dot{m}(h_1 - h_4) = \frac{6}{60} \times (244.51 - 90.69) = \boxed{15.382\ kW}$

d) $COP = \frac{\dot{Q}_L}{\dot{W}_{comp}} = \frac{15.382}{2.6678} = \boxed{5.766}$

-) $v = \frac{RT}{P} - \alpha \rightarrow \left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P}$

$$\Rightarrow \mu_{JT} = \frac{-1}{c_p}\left[v - \frac{T \times R}{P}\right] = \frac{-1}{c_p}\left[v - (v + \alpha)\right] = \frac{\alpha}{c_p}$$

since both α and c_p are $> 0 \rightarrow \mu_{JT} > 0$

⇒ During the throttling process, the temperature of the refrigerant will <u>decrease</u>

f) $h_{fg} = T \, v_{fg} \times \left(\frac{dP}{dT}\right)_{sat@-10°C}$

but $v_{fg} = v_g - v_f = 0.098516 - 0.0007535 = 0.0987625 \; m^3/Kg$

and $\left(\frac{dP}{dT}\right)_{sat@-10°C} \simeq \left(\frac{\Delta P}{\Delta T}\right)_{sat@-10°C} = \frac{P_{sat@-8°C} - P_{sat@-12°C}}{-8-(-12)}$

$= \frac{217.08 - 185.37}{4} = 7.9275 \; \frac{KPa}{K}$

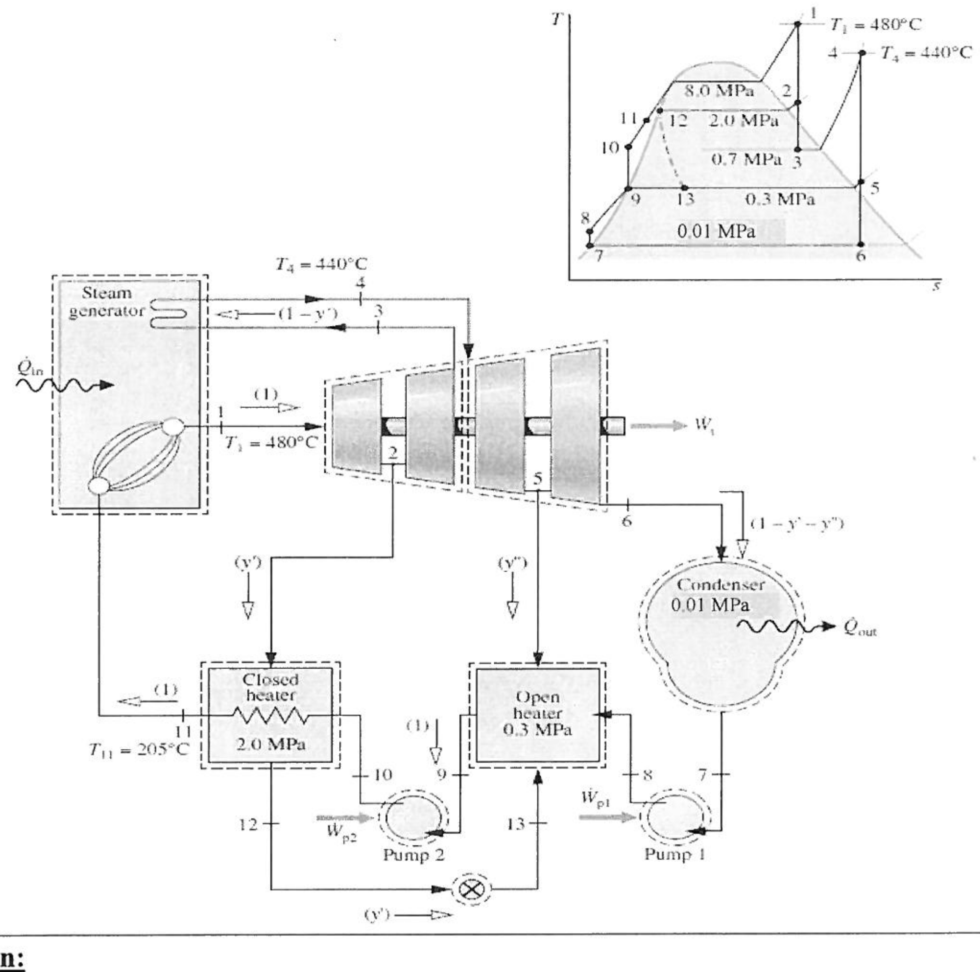
$\Rightarrow h_{fg@-10°C} = 263.15 \times 0.0987625 \times 7.9275 = \boxed{206.031 \; KJ/Kg}$

the tabulated value is 205.96 KJ/Kg which is very close to the calculated value.

Problem II (30 points)

Consider a reheat–regenerative vapor power cycle with two feedwater heaters, a closed feedwater heater and an open feedwater heater. The cycle and its corresponding *T-s* diagram are shown below. Steam enters the first turbine at 8.0 MPa, 480°C and expands to 0.7 MPa. The steam is reheated to 440°C before entering the second turbine, where it expands to the condenser pressure of 0.01 MPa. Steam is extracted from the first turbine at 2 MPa and fed to the closed feedwater heater. Feedwater leaves the closed heater at 205°C and 8.0 MPa while the steam extracted from the turbine exits the closed feedwater heater as saturated liquid at 2MPa which is then throttled into the open feedwater heater. Steam extracted from the second turbine at 0.3MPa is also fed into the open feedwater heater, which operates at 0.3 MPa. The stream exiting the open feedwater heater is saturated liquid at 0.3 MPa. The net power output of the cycle is 100 MW. Determine:

a) The specific enthalpy *h* of the water at states 1 to 13 (**<u>13 points</u>**).
b) The fractions of steam y' and y'' extracted from the turbine (**<u>7 points</u>**).
c) The thermal efficiency (**<u>7 points</u>**).
d) The mass flow rate of the steam entering the first turbine, in **<u>kg/h</u>** (**<u>3 points</u>**).



<u>Solution:</u>

Solution:

a) First let's determine the properties at all points:

@ ①: $P_1 = 8\,MPa$, $T_1 = 480°C$ $\rightarrow$ $h_1 = 3349.02\ KJ/Kg$ & $s_1 = 6.65912\ KJ/Kg.K$

@ ②: $P_2 = 2\,MPa$, $s_2 = s_1 = 6.65912\ KJ/Kg.K$ $\rightarrow$ $h_2 = 2964.39\ KJ/Kg.$

@ ③: $P_3 = 0.7\,MPa$, $s_3 = s_1 = 6.65912\ KJ/Kg.K$ $\rightarrow$ $x_3 = \frac{s_3 - s_f}{s_{fg}} = \frac{6.65912 - 1.9918}{4.7153} = 0.989825$

$h_3 = h_f + x_3 h_{fg} = 697 + 0.989825 \times 2065.8$

$\Rightarrow h_3 = 2741.78\ KJ/Kg$

@ ④: $P_4 = 0.7\,MPa$, $T_4 = 440°C$ $\rightarrow$ $h_4 = 3354.49\ KJ/Kg$ & $S_4 = 7.75962\ KJ/Kg.K$

@ ⑤: $P_5 = 0.3\,MPa$, $S_5 = S_4 = 7.75962\ KJ/Kg.K$ $\rightarrow$ $h_5 = 3104.385\ KJ/Kg$

@ ⑥: $P_6 = 0.01\ MPa$, $S_6 = S_5 = 7.75962\ KJ/Kg.K$ $\rightarrow$ $X_6 = \frac{S_6 - S_f}{S_{fg}} = \frac{7.75962 - 0.6492}{7.4996} = 0.9481$

$\Rightarrow h_6 = h_f + x_6 h_{fg} = 191.81 + 0.9481 \times 2392.1$

$h_6 = 2459.76\ KJ/Kg$

@ ⑦: $h_7 = h_f @ 0.01MPa = 191.81\ KJ/Kg$

@ ⑧: $h_8 = h_7 + v_7 (P_8 - P_7) = 191.81 + 0.001010 \times (0.3 - 0.01) \times 10^3 = 192.1\ KJ/Kg$

@ ⑨: $h_9 = h_f @ 0.3MPa = 561.43\ KJ/Kg$

@ ⑩: $h_{10} = h_9 + v_9 (P_{10} - P_9) = 561.43 + 0.001073 \times (8000 - 300) = 569.69\ KJ/Kg$

@ ⑪: $P_{11} = 8\,MPa$, $T_{11} = 205°C$ $\rightarrow$ $h_{11} = 877.519\ KJ/Kg$

@ ⑫: $h_{12} = h_f @ 2MPa = 908.47\ KJ/Kg$

@ ⑬: $h_{13} = h_{12} = 908.47\ KJ/Kg$

b) Energy balance:

Open FWH: $y'' h_5 + (1-y'-y'') h_8 + y' h_{13} = h_9$ ①

Closed FWH: $h_{11} - h_{10} = y' (h_2 - h_{12})$ ②

② → $y' = \dfrac{h_{11} - h_{10}}{h_2 - h_{12}} = \dfrac{877.519 - 563.69}{2964.39 - 908.47} = \boxed{0.14973}$

① ⇒ $y'' \times 3104.385 + (1 - 0.14973 - y'') \times 192.1 + 0.14973 \times 908.47 = 561.43$

$2912.285 y'' = 262.0679$

$\boxed{y'' = 0.089987}$

c) $\eta_{th} = \dfrac{w_{net,out}}{q_{in}}$

$= \dfrac{(h_1 - h_2) + (1-y')(h_2 - h_3) + (1-y')(h_4 - h_5) + (1-y'-y'')(h_5 - h_6) - (1-y'-y'')(h_8 - h_7) - (h_{10} - h_9)}{(h_1 - h_{11}) + (1-y')(h_4 - h_3)}$

$= \dfrac{(3349.02 - 2964.39) + (1 - 0.14973)(2964.39 - 2741.78 + 3354.49 - 3104.385) + (1 - 0.14973 - 0.089987)(3104.385 - 2459.76 - 192.1 + 191.81) - (569.69 - 561.43)}{(3349.02 - 877.519) + (1 - 0.14973)(3354.49 - 2741.78)}$

$= \dfrac{384.63 + 401.9354 + 489.877 - 8.26}{2471.501 + 520.9689} = \dfrac{1268.1824}{2992.4699} = 0.4238$

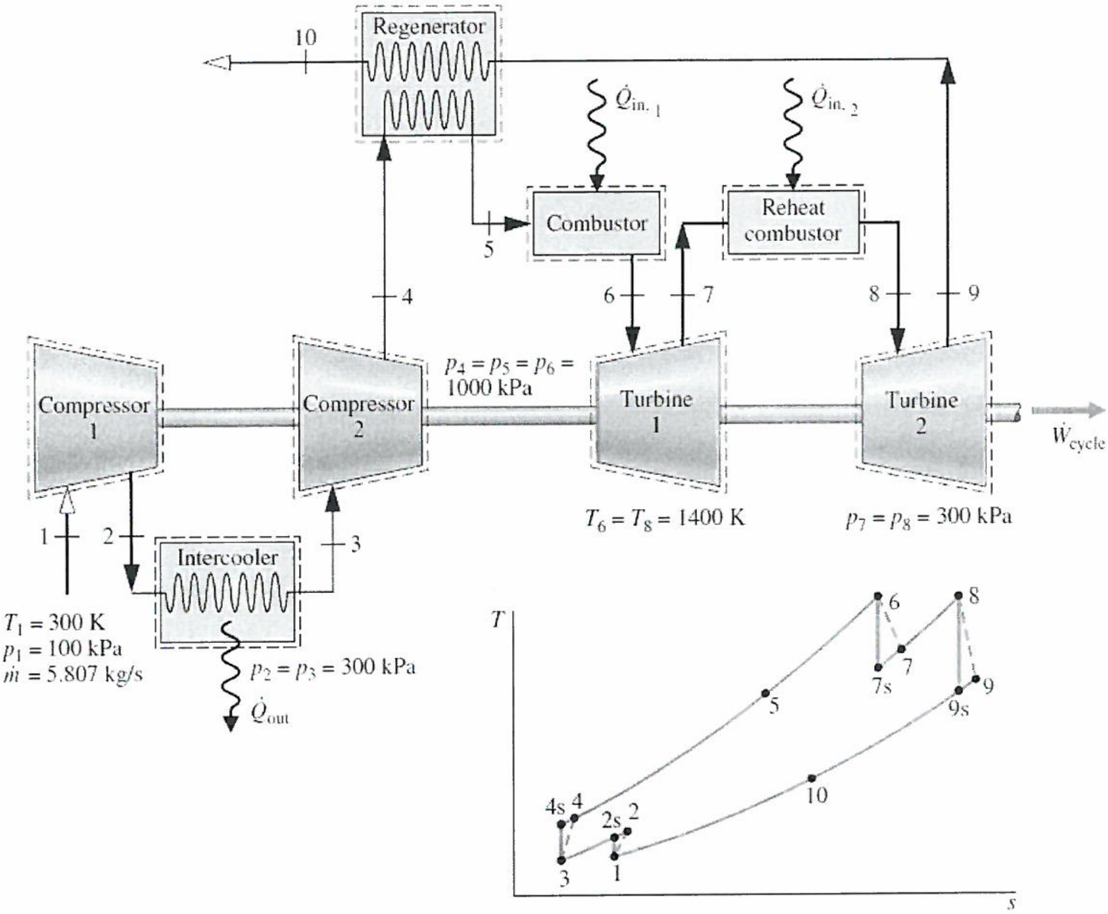
$= \boxed{42.38\%}$

d) $\dot{W}_{net} = \dot{m}\, w_{net}$

⇒ $\dot{m} = \dfrac{\dot{W}_{net}}{w_{net}} = \dfrac{100 \times 10^3}{1268.1824} = 78.853 \text{ kg/s} = \boxed{283870.84 \text{ kg/h}}$

Problem III (25 points)

A regenerative gas turbine with intercooling and reheat operates at steady state. The corresponding cycle and *T-s* diagram are shown below:



Air enters the compressor at 100 kPa, 300 K with a mass flow rate of 5.807 kg/s. The pressure ratio across the two-stage compressor is 10. The pressure ratio across the two-stage turbine is also 10. The intercooler and reheater each operate at 300 kPa. At the inlets to the turbine stages, the temperature is 1400 K. The temperature at the inlet to the second compressor stage is 300 K. The isentropic efficiency of each compressor and turbine stage is 80%. The regenerator effectiveness is 80%. Determine:

a) The specific enthalpy *h* of Air at states 1 to 10 (**10 points**).

b) The thermal efficiency (**5 points**).

c) The back work ratio (**5 points**).

d) The net power developed, in kW (**5 points**).

Solution:

a) @ ①: $T_1 = 300\text{ K} \rightarrow \boxed{h_1 = 300.19\text{ kJ/kg}}$ & $P_{r_1} = 1.386$

$$\frac{P_{r_2}}{P_{r_1}} = \frac{P_2}{P_1} = \frac{300}{100} = 3 \rightarrow P_{r_2} = 3 \times P_{r_1} = 3 \times 1.386 = 4.158$$

$$\rightarrow \boxed{h_{2s} = 411.257\text{ kJ/kg}}$$

but $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = 0.8 \rightarrow h_2 = \frac{h_{2s} - h_1}{0.8} + h_1 = \frac{411.257 - 300.19}{0.8} + 300.19$

$\boxed{h_2 = 439.024 \text{ KJ/Kg}}$

@③: $T_3 = 300K \rightarrow \boxed{h_3 = 300.19 \text{ KJ/Kg}}$ & $P_{r_3} = 1.386$

$\frac{P_{r_4}}{P_{r_3}} = \frac{P_4}{P_3} = \frac{1000}{300} = 3.333 \rightarrow P_{r_4} = \frac{10}{3} \times 1.386 = 4.62$

$\rightarrow \boxed{h_{4s} = 423.796 \text{ KJ/Kg.}}$

but $h_4 = \frac{h_{4s} - h_3}{0.8} + h_3 = \frac{423.796 - 300.19}{0.8} + 300.19$

$\rightarrow \boxed{h_4 = 454.6975 \text{ KJ/Kg}}$

The effectiveness of the regenerator ε is:

$\boxed{\varepsilon = \frac{h_5 - h_4}{h_9 - h_4} = 0.8}$ (*)

@⑥: $T_6 = 1400K \rightarrow \boxed{h_6 = 1515.42 \text{ KJ/Kg}}$ & $P_{r_6} = 450.5$

$\frac{P_{r_7}}{P_{r_6}} = \frac{P_7}{P_6} = \frac{300}{1000} \rightarrow P_{r_7} = 0.3 \times 450.5 = 135.15$

$\rightarrow \boxed{h_{7s} = 1095.866 \text{ KJ/Kg}}$

but $\eta_T = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.8 \rightarrow h_7 = h_6 - 0.8(h_6 - h_{7s})$

$= 1515.42 - 0.8(1515.42 - 1095.866)$

$\rightarrow \boxed{h_7 = 1179.7768 \text{ KJ/Kg}}$

@⑧: $T_8 = 1400K \rightarrow \boxed{h_8 = 1515.42 \text{ KJ/Kg}}$ & $P_{r_8} = 450.5$

$\frac{P_{r_8}}{P_{r_9}} = \frac{P_8}{P_9} = \frac{300}{100} = 3 \rightarrow P_{r_9} = \frac{P_{r_8}}{3} = \frac{450.5}{3} = 150.17$

$\rightarrow \boxed{h_{9s} = 1127.64 \text{ KJ/Kg}}$

but $\eta_T = \frac{h_8 - h_9}{h_8 - h_{9s}} = 0.8 \rightarrow h_9 = h_8 - 0.8(h_8 - h_{9s})$

$h_9 = 1515.42 - 0.8(1515.42 - 1127.64)$

$\boxed{h_9 = 1205.196\ KJ/Kg}$

(*) $\Rightarrow h_5 = 0.8(h_9 - h_4) + h_4 = 0.8(1205.196 - 454.6875) + 454.6875$

$\boxed{h_5 = 1055.0963\ KJ/Kg}$

Finally; Energy balance of regenerator:

$h_9 - h_{10} = h_5 - h_4 \rightarrow h_{10} = h_9 - h_5 + h_4$

$= 1205.196 - 1055.0963 + 454.6875$

$\boxed{h_{10} = 604.7872\ KJ/Kg}$

b) $\eta_{th} = ?$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{(h_6 - h_7) + (h_8 - h_9) - (h_4 - h_3) - (h_2 - h_1)}{(h_6 - h_5) + (h_8 - h_7)}$$

$$= \frac{(1515.42 - 1179.7768) + (1515.42 - 1205.196) - (454.6875 - 300.19) - (439.024 - 300.19)}{(1515.42 - 1055.0963) + (1515.42 - 1179.7768)}$$

$$= \frac{(335.6232 + 310.224) - (154.5075 + 138.834)}{460.3237 + 335.6432} = \frac{645.8472 - 293.3415}{795.9669}$$

$\Rightarrow \eta_{th} = 0.442865 = \boxed{44.2865\ \%}$

c) $r_{bw} = \frac{w_{comp}}{w_{turb}} = \frac{(h_2 - h_1) + (h_4 - h_3)}{(h_6 - h_7) + (h_8 - h_9)} = \frac{293.3415}{645.8472} = 0.4542 = \boxed{45.42\ \%}$

d) $\dot{W}_{net} = \dot{m} \times w_{net} = \dot{m} \times (645.8472 - 293.3415) = 5.807(645.8472 - 293.3415)$

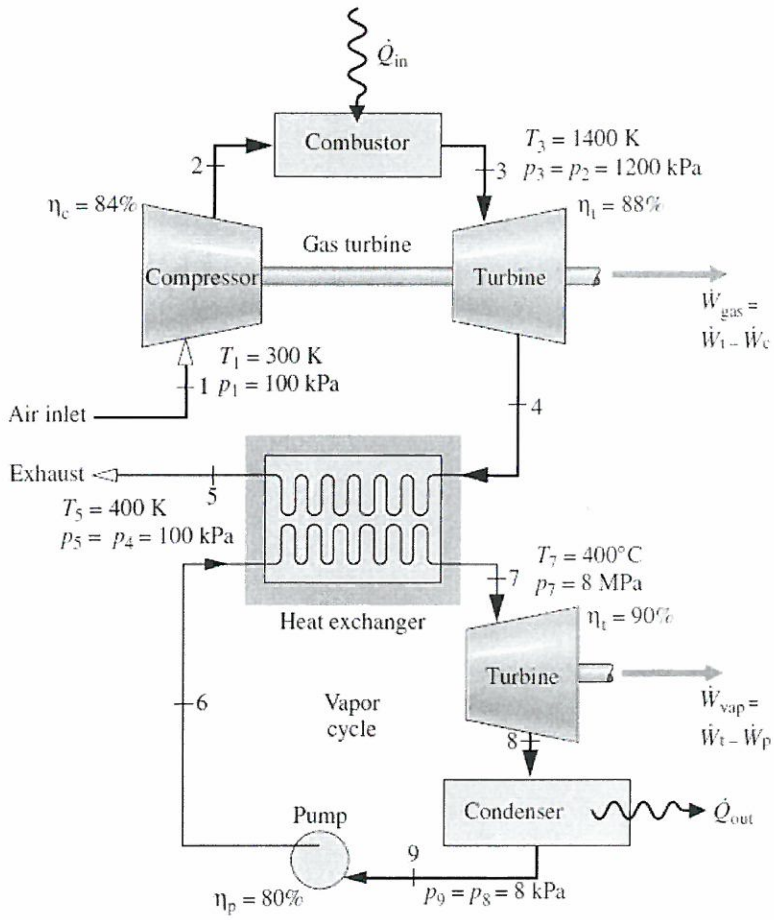
$\boxed{\dot{W}_{net} = 2047\ Kw}$

Problem IV (15 points)

The combined gas turbine–vapor power plant shown below has a net power output of 45 MW. Air enters the compressor of the gas turbine at 100 kPa, 300 K, and is compressed to 1200 kPa. The isentropic efficiency of the compressor is 84%. The condition at the inlet to the turbine is 1200 kPa, 1400 K. Air expands through the turbine, which has an isentropic efficiency of 88%, to a pressure of 100 kPa. The air then passes through the interconnecting heat exchanger and is finally discharged at 400 K. Steam enters the turbine of the vapor power cycle at 8 MPa, 400°C, and expands to the condenser pressure of 8 kPa. Water enters the pump as saturated liquid at 8 kPa. The turbine and pump of the vapor cycle have isentropic efficiencies of 90 and 80%, respectively. Determine:

a) The mass flow rates of the air and the steam, each in kg/s, and the net power developed by the gas turbine and vapor power cycle, each in MW (**10 points**).

b) The thermal efficiency of this combined cycle (**5 points**).



Solution:

a) $T_1 = 300K \rightarrow$ $h_1 = 300.19$ kJ/kg & $P_{r_1} = 1.386$

$\frac{P_{r_2}}{P_{r_1}} = \frac{P_2}{P_1} = \frac{1200}{00} = 12 \rightarrow P_{r_2} = 12 P_{r_1} = 12 \times 1.386 = 16.632$

$\rightarrow h_{2s} = 610.65$ kJ/kg.

but $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = 0.84 \rightarrow h_2 = \frac{h_{2s} - h_1}{0.84} + h_1$

$h_2 = \frac{610.65 - 300.19}{0.84} + 300.19$

$\boxed{h_2 = 669.785 \text{ KJ/Kg}}$

@③: $T_3 = 1400K \rightarrow \boxed{h_3 = 1515.42 \text{ KJ/Kg}}$ & $P_{r_3} = 450.5$

$\frac{P_{r_3}}{P_{r_4}} = \frac{P_3}{P_4} = \frac{1200}{100} = 12 \rightarrow P_{r_4} = \frac{P_{r_3}}{12} = \frac{450.5}{12} = 37.54$

$\Rightarrow h_{4s} = 768.37 \text{ KJ/Kg}$

but $\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = 0.88 \rightarrow h_4 = h_3 - 0.88(h_3 - h_{4s})$

$= 1515.42 - 0.88(1515.42 - 768.37)$

$\rightarrow \boxed{h_4 = 858.016 \text{ KJ/Kg}}$

@⑤: $T_5 = 400K \rightarrow \boxed{h_5 = 400.98 \text{ KJ/Kg}}$

@⑦: $\left.\begin{array}{l} T_7 = 400°C \\ P_7 = 8MPa \end{array}\right\} \rightarrow \boxed{h_7 = 3139.4 \text{ KJ/Kg}}$

@⑧: $\left.\begin{array}{l} P_8 = 8KPa \\ \text{sat. Liq.} \end{array}\right\} \rightarrow h_8 = h_f @ 8KPa = \boxed{173.362 \text{ KJ/Kg}}$

$h_{6s} = h_8 + v_8 (P_6 - P_8) = 173.362 + 0.0010084(8000 - 8)$

$\Rightarrow h_{6s} = 181.42 \text{ KJ/Kg}$

but $\eta_P = \frac{h_{6s} - h_8}{h_6 - h_8} = 0.8 \rightarrow h_6 = \frac{h_{6s} - h_8}{0.8} + h_8 = \frac{181.42 - 173.362}{0.8} + 173.362$

$\Rightarrow \boxed{h_6 = 183.4345 \text{ KJ/Kg}}$

@ (8s): $P_{8s} = 8\,kPa$, $s_8 = s_7 = 6.3658\ kJ/kg.K$ → @ 8 kPa: $s_f = 0.59088\ kJ/kg.K$, $s_g = 8.2288\ kJ/kg.K$

$$\Rightarrow x_{8s} = \frac{6.3658 - 0.59088}{8.2288 - 0.59088} = 0.75598$$

$$\Rightarrow h_{8s} = h_f + x_{8s} h_{fg} = 173.362 + 0.756 \times 2402.66 = 1989.77\ kJ/kg$$

but $\eta_T = \frac{h_7 - h_8}{h_7 - h_{8s}} = 0.9 \longrightarrow h_8 = h_7 - 0.9(h_7 - h_{8s})$

$$h_8 = 3139.4 - 0.9 \times (3139.4 - 1989.77)$$

$$\boxed{h_8 = 2104.73\ kJ/kg}$$

En. balance on heat exchanger:

$$\dot m_{air}(h_4 - h_5) = \dot m_w (h_7 - h_6)$$

$$\Rightarrow \frac{\dot m_{air}}{\dot m_w} = \frac{h_7 - h_6}{h_4 - h_5} = \frac{3139.4 - 183.4345}{858.016 - 400.98} = \boxed{6.4677}$$

And $\dot W_{net} = 45\,MW = \dot m_{air}(h_3 - h_4 - h_2 + h_1) + \dot m_w (h_7 - h_8 - h_6 + h_9)$

$$45 \times 10^3 = 6.4677\, \dot m_w (1515.42 - 858.016 - 669.785 + 300.19) + \dot m_w (3139.4 - 2104.73 - 183.4345 + 173.362)$$

$$45 \times 10^3 = 6.4677 \times \dot m_w \times 287.81 + \dot m_w \times 1024.5975$$

$$\Rightarrow \boxed{\dot m_w = 15.71\ kg/s} \quad \text{and} \quad \boxed{\dot m_{air} = 6.4677 \times 15.71 = 100.84\ kg/s}$$

$$\dot W_{net,gas} = 6.4677 \times \dot m_w \times 287.81 = 29240.29\ kW \simeq \boxed{29\,MW}$$

$$\Rightarrow \dot W_{net,vapor} = \dot m_w \times 1024.5975 = 15936.47\ kW \simeq \boxed{16\,MW} = 45 - \dot W_{net,gas}$$

b) $$\eta_{th} = \frac{\dot W_{net}}{\dot Q_{in}} = \frac{45 \times 10^3}{\dot m_{air} \times (h_3 - h_2)} = \frac{45000}{100.84 \times (1515.42 - 669.785)} = 0.5277 = \boxed{52.77\%}$$